A NOTE ON MODIFIED RATIO ESTIMATOR USING TRANSFORMATION

By

Miss S. P. Kulkarni

University of Poona

(Received : March, 1977)

1. Introduction

The precision in estimating the population parameters of a finite population may be increased by using an auxiliary variate X, which is highly correlated with Y, the character under study. The use of the ratio estimator is advisable when the line of regression of Y on X passes through the origin. With the knowledge of the intercept made by the regression line with the Y axis, we can modify the usual ratio estimator. Considering this Mohanty and Das [1], modified the usual ratio estimator. In this note, we compare the absolute biases of the modified ratio estimator and the usual ratio estimator and obtain the regions in which the modified ratio estimator has less absolute bias than the usual ratio estimator.

2. Notation and Preliminaries

From the population of size N, we draw a sample of size n, by the method of simple random sampling without replacement. Let $\bar{X}$, $\bar{Y}$ be the population means and $\bar{x}$, $\bar{y}$ be the sample means for the respective characteristics. Let X_i's be positive and the sample size be sufficiently large. Let $Y=\alpha+\beta X$ be the line of regression for the population. Throughout this paper we will take α and β to be positive.

Let a and b be the estimators of α and β respectively, obtained through the pilot survey. We take a and b to be positive.

Define the modified auxiliary variate as $X_t=X+\frac{a}{b}$.

The usual ratio estimator for the population mean is

$$\bar{y}_R=\frac{\bar{y}}{\bar{x}}\times\bar{X}$$

Then the modified ratio estimator is

$$\bar{y}_t = \frac{\bar{y}}{\bar{x}_t}\,\bar{X}_t$$

Assuming the terms of second order and higher order to be negligible we can write, for the bias in $\bar{y}_R$, as

$$B(\bar{y}_R) = \frac{\theta\alpha\, S_x^2}{\bar{X}^2}$$

where $$\theta = \frac{1}{n} - \frac{1}{N}$$

and $$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$$

we write

$$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})$$

Now

$$B(\bar{y}_t) = \theta\left[\alpha - \beta\,\frac{a}{b}\right]\frac{S_x^2}{\bar{X}_t^2}$$

3. RESULTS

It is clear that if $B(\bar{y}_t)$ is positive, then $\bar{y}_t$ has less bias (and therefore less absolute bias) than $\bar{y}_R$. We shall see what happens when $B(\bar{y}_t)$ is negative.

Lemma 1. If

(*i*) $B(\bar{y}_t) < 0$,

(*ii*) $\bar{X}^2 - 4\,\frac{\alpha}{\beta}\,\bar{X} - 4\,\frac{\alpha^2}{\beta^2} < 0$

then $\bar{y}_t$ has less absolute bias than $\bar{y}_R$.

Proof :

$$| B(\bar{y}_R) | - | B(\bar{y}_t) |$$

$$= \theta\beta S_x^2\left[\bar{X}\left(\bar{X} + \frac{a}{b}\right)\right]^{-2}\left[\frac{\alpha}{\beta}\,\frac{a^2}{b^2} + \left(2\,\frac{\alpha}{\beta} - \bar{X}\right)\frac{\bar{X}a}{b} + 2\frac{\alpha}{\beta}\bar{X}^2\right]$$

Now treating

$$\frac{\alpha}{\beta}\,\frac{a^2}{b^2} + \left(2\,\frac{\alpha}{\beta} - \bar{X}\right)\bar{X}\,\frac{a}{b} + 2\,\frac{\alpha}{\beta}\bar{X}^2$$

as a quadratic in $\frac{a}{b}$, we see that if

$$\bar{X}^2 - 4\frac{\alpha}{\beta}\bar{X} - 4\frac{\alpha^2}{\beta^2} < 0$$

then $\bar{y}_t$ has less absolute bias than $\bar{y}_R$.

Lemma 2 : If

(*i*) $B(\bar{y}_t) < 0$,

(*ii*) $\bar{X}^2 - 4\frac{\alpha}{\beta}\bar{X} - 4\frac{\alpha^2}{\beta^2} > 0$

and (*iii*) $\frac{a}{b}$ does not lie in the closed interval $[\xi_1, \xi_2]$

where

$$\xi_1 = \frac{\beta}{2\alpha}\left[\bar{X}^2 - 2\frac{\alpha}{\beta}\bar{X} - \bar{X}\sqrt{\bar{X}^2 - 4\frac{\alpha}{\beta}\bar{X} - 4\frac{\alpha^2}{\beta^2}}\right]$$

$$\xi_2 = \frac{\beta}{2\alpha}\left[\bar{X}^2 - 2\frac{\alpha}{\beta}\bar{X} + \bar{X}\sqrt{\bar{X}^2 - 4\frac{\alpha}{\beta}\bar{X} - 4\frac{\alpha^2}{\beta^2}}\right]$$

then $\bar{y}_t$ has less absolute bias than $\bar{y}_R$.

Proof :

$$|B(\bar{y}_R)| - |B(\bar{y}_t)|$$

$$= \theta\beta\, S_x^2 \left[\bar{X}\left(\bar{X} + \frac{a}{b}\right)\right]^{-2} \left[\frac{\alpha}{\beta}\frac{a^2}{b^2} + \left(2\frac{\alpha}{\beta} - \bar{X}\right)\frac{a}{b}\bar{X} + \frac{2\alpha}{\beta}\bar{X}^2\right]$$

$$= \theta\alpha\, S_x^2 \left[\bar{X}\left(\bar{X} + \frac{a}{b}\right)\right]^{-2}\left(\frac{a}{b} - \xi_1\right)\left(\frac{a}{b} - \xi_2\right) > 0,$$

where ξ_1 and ξ_2 are as defined abave.

Hence the lemma.

Theorem : (*I*) Suppose $B(\bar{y}_t) > 0$. In this case $\bar{y}_t$ has less absolute bias than $\bar{y}_R$.

(*II*) Suppose $B(\bar{y}_t) < 0$. In this case $\bar{y}_t$ has less absolute bias than $\bar{y}_R$ provided,

(*i*) $\bar{X}^2 - 4\frac{\alpha}{\beta}\bar{X} - 4\frac{\alpha^2}{\beta^2} < 0$ or

(*ii*) $\bar{X}^2 - 4\frac{\alpha}{\beta}\bar{X} - 4\frac{\alpha^2}{\beta^2} > 0$ and $\frac{a}{b}$

does not lie in the closed interval $[\xi_1, \xi_2]$ where ξ_1 and ξ_2 are as defined in the lemma 2.

The proof is quite straightforward with the use of the lemmas 1 and 2.

SUMMARY

Knowledge about the regression line can be utilized to modify the usual ratio estimator. In this note regions are obtained where the modified ratio estimator (suggested in [1]) has less absolute bias than the usual ratio estimator.

ACKNOWLEDGEMENT

I am grateful to Dr. A. V. Kharshikar, University of Poona, for the keen interest he had taken in the preparation of this note and to the referees for their valuable suggestions. Thanks are also due to University Grants Commission, New Delhi, for awarding me a research Scholarship.

REFERENCE

[1] Mohanty, S. and Das, M. N. [1971] : Use of transformation in sampling, *Jour. Ind. Soc. Agri. Stat.*, Vol. 23, 83-87.